REPRISE

What we have learned about systems with LARGE NUMBER OF PARTICLES

1) Temp $T \to$ Kinetic energy of molecules

$kT \sim$ ENERGY per dof. in A CLASSICAL GAS

$\downarrow$

EQUIPARTITION.

2) ENERGY CONSERVATION IN a SYSTEM in thermodynamic Equilibrium.

1ST LAW $\quad \Delta U = W + Q$

energy change

work done

Heat

IN ALL EXPERIMENTS

— heat flows from hot → cold.

— gas diffuses to fill allowed volume

↳ NEVER Comes BACK

→ BOTH these processes happen SPONTANEOUSLY

ALWAYS in Same Direction

→ IT takes work (= $$) to move

heat from COLD → HOT

AIR CONDITIONING ⟶ Hydro Bill.

WHY do experiments give these results?

IN 19^{TH}, 20^{TH} CENTURY BOLTZMANN / Gibbs

systems with very large numbers of particles → QUALITATIVELY Diffent from small numbers.

↳ phenomena discussed happen in preferred direction

↳ Explained by RANDOMNESS of DISTRIB OF ENERGY AMONG many particles in large N SYSTEM

ALL thermodynamics + Stat Mech FOLLOW

FROM one POSTULATE

⇒ IN A CLOSED (isolated, fixed energy)

system all accessible MICROSTATES

are EQUALLY LIKELY IN TD Equilib

↳ NO PROOF OF THIS.

— in isolated gas, total energy E

— enormous number of ways to

distribute energy among

many N particles

- collisions continually change the way energy is distributed
  - ↳ MICRO STATE changes continuously
  - ↳ SYSTEM continually in DIFFERENT MICROSTATES
  - ↳ $\bar{v}_i$ change continuously.
- in TD Equilibrium any such state is as likely as any OTHER.
- DETAILED MECHANICAL Description

↓

Abandoned

↓

talk about Averages probabilities

AVERAGES
PROBABILITIES $\Big\}$ STATISTICAL MECHANICS

↓

appropriate to TD Equilibrium

↓ MACROSCOPIC PARAMETERS STATIC

ARGUMENT in favour of STAT MECH

↳ IT works

↳ describes known phenomena

↳ makes predictions

COSMIC MICROWAVE BG

BOSE-EINSTEIN CONDENSATE

TOPOLOGICAL PHASES

⟹ SO NOW WE go on to STAT MECH

IDEAL GAS ?

Seems simple — complex to count

AVAILABLE # OF MICROSTATES.

↳ things are moving around, vibrating

rotating → complex system.

↳ How many ways can you

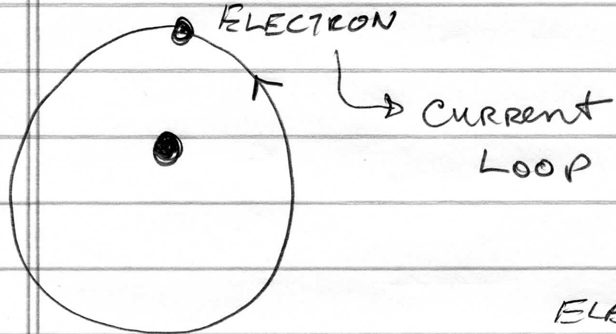
arrange all these things in 10^{23}

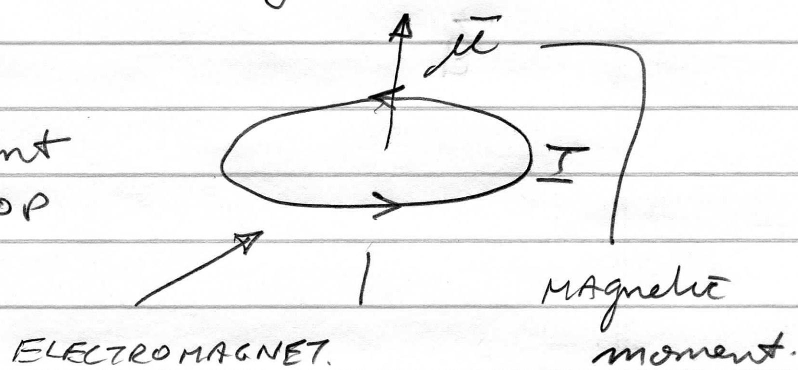
particles

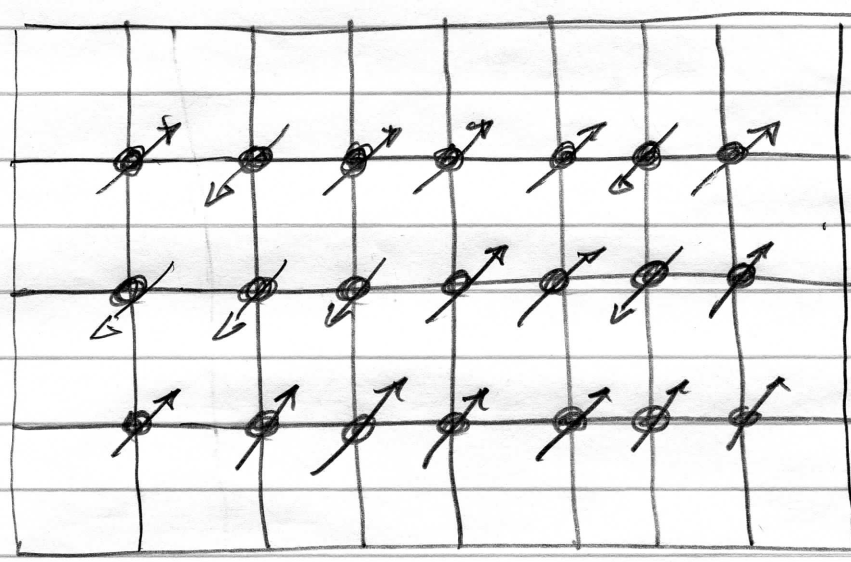
↳ A LOT

— START WITH something SIMPLER

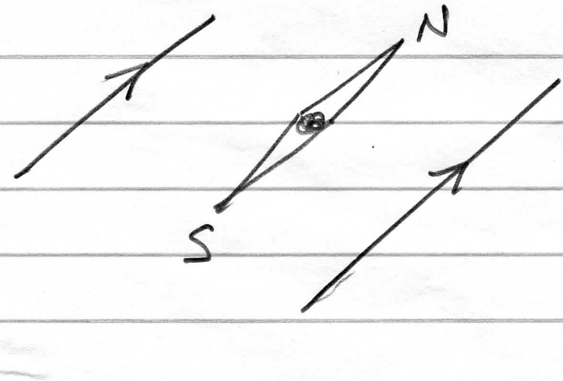
PARAMAGNET — CRYSTAL, all atoms have angular momentum







Think about hiking compass NOT GPS!



IN EXTernal $\bar{B}$ field

↳ arrow orients itself ALONG $\bar{B}$

If arrow not INITIALLY $\parallel^{el}$ $\bar{B}$

↳ Rotates to Align

$\therefore$ TORQUE acting on ARROW

↳ something does work

↳ changes ENERGY.

$$V = \int_{90^\circ}^{\theta} \tau \, d\theta \qquad \tau = \mu B \sin\theta$$

$$= \mu B \int_{90^\circ}^{\theta} \sin\theta \, d\theta$$

$$= -\mu B \cos\theta$$

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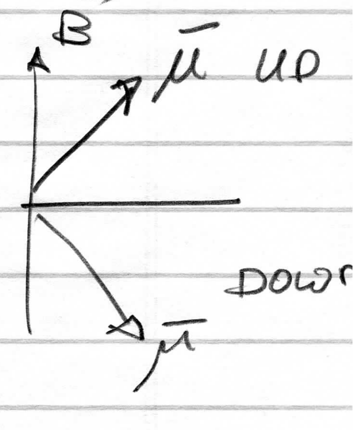
→ magnetic moment is QUANTUM COMPASS

↳ values of angular momentum quantized

↳ values of magnetic moment $\bar{\mu}$ quantized

↳ Energy in $\bar{B}$ field quantized



Actually → multi electron atoms

→ $\bar{\mu}$ comes from electron in unfilled outer shell

→ ELECTRONIC PARAMAGNET

$$\bar{\mu} = \mu_0 \bar{S} \leftarrow \hbar$$

spin

Let's use units $S = \pm 1$

S is dimensionless in our units.

$$[\mu_0] = \left[\frac{\text{ENERGY}}{\text{B FIELD}}\right]$$

? What is energy of Magnetic Moment in External B field

$$U = -\bar{\mu} \cdot \bar{B}$$

energy minimum when $\bar{\mu} \cdot \bar{B}$ maximum.

$$\bar{\mu} \cdot \bar{B} = |\bar{\mu}||\bar{B}|\cos\theta$$

$$\cos\theta \rightarrow 1 \qquad \theta \rightarrow 0$$

$\bar{\mu} \cdot \bar{B}$ minimized for $\theta = 0$

If ROTATE $\bar{\mu}$ antiparallel

↳ have to do work against $\bar{B}$

↳ raise energy of magnetic moment

For a single spin

$$U = -\mu_0 B S \qquad S = \begin{matrix} +1 & \uparrow \\ -1 & \downarrow \end{matrix}$$

for N spins $S_1 \text{------} S_N$ all ± 1

energy $U = -\mu_0 B \underbrace{(S_1 + S_2 \text{----} S_N)}_{\text{TOTAL SPIN } S_T}$

How does this system come into thermoDynamic Equilibrium?

↳ like ideal gas — atoms in lattice exchange energy

NOT BY COLLISIONS

↳ some kind of interaction between spins

↳ atomic vibrations.

↳ each spin produces tiny $\bar{B}$ field

↳ seen by other spins.

Don't need to understand details

↳ only need to assume that there is some interaction which brings system of N spins into thermodynamic Equilibrium

STATE MECH postulate is that TD Equilib

Corresponds to every ACCESSIBLE

MICROSTATE being EQUALLY LIKELY

Actually — what is a MICROSTATE?

$+1 \quad -1 \quad -1 \quad +1 \quad -1 \quad \cdots\cdots \quad N$

$\uparrow \quad \downarrow \quad \downarrow \quad \uparrow \quad \downarrow \quad - - - - -$

Analagous to specifying $\vec{r}, \vec{p}$ of

N particles in gas.

What is a MACROSTATE?

$\hookrightarrow$ macroscopic variable $S_T = S_1 + \cdots + S_N$

S_T goes from $-N$ all $\downarrow$

$+N$ all $\uparrow$

$\uparrow \uparrow \downarrow \uparrow \uparrow \downarrow$

Flipping AN INDIVIDUAL SPIN $\downarrow \longrightarrow \uparrow$

CHANGES S_T by 2 units

Say for $N=2$

$$\left.\begin{array}{cc} & S \\ \uparrow\uparrow & 2 \\ \uparrow\downarrow & 0 \\ \downarrow\downarrow & -2 \end{array}\right\} \begin{array}{l} \text{\# OF} \\ \text{MACRO STATES} \\ = 3 \\ = N+1 \end{array}$$

Imagine STARTING ALL UP

		S
	$\uparrow \uparrow \uparrow \uparrow \uparrow$ - - - - - - - N	0
flip 1	$\downarrow \uparrow \uparrow \uparrow \uparrow$	1
flip 100		100

NUMBER OF MACROSTATES $= N+1$

$\hookrightarrow$ all have Different Energy S_T

How many MICROSTATES ?

↳ I.E. How many possible arrangements of INDIVIDUAL SPINS ?

$$\underset{2^N}{\uparrow} \qquad \left(2^{10^{23}} \,! \right)$$

Each spin Corresponds — 2 microstates

$$\underbrace{2 \times 2 \times 2 \text{ --------- } 2}_{N \text{ times}}$$

STAT MECH — FOR an ISOLATED SYSTEM
↳ U fixed

Every accessible MICROstate is EQUALLY LIKELY

$$U \rightarrow \text{fixed}, \; U = -\mu_0 B S_T$$

S_T fixed

↓

MACROSTATE

Accessible MICROSTATES

↳ THOSE that have S_T, out of the possible 2^N.

S_T depends on $\Sigma\uparrow$ and $\Sigma\downarrow$

$$S_T = \underbrace{N_\uparrow}_{\Sigma\uparrow} - \underbrace{N_\downarrow}_{\Sigma\downarrow}$$

But $N_\uparrow + N_\downarrow = N \leftarrow$ TOTAL.

So $N_\downarrow = N - N_\uparrow$

$$S_T = N_\uparrow - N + N_\uparrow$$

$$S_T = -N + 2N_\uparrow$$

Can label a MICROSTATE BY S, $N_\downarrow$, or $N_\uparrow$.

LET'S choose $N_\uparrow$, makes sense since

$$N_\uparrow = 0, 1, 2, \ldots\ldots N-1, N$$

makes clear that there are $N+1$ MACROSTATES

MOST IMPORTANT attribute of a STAT MECH STATE which is isolated is

— MULTIPLICITY OF A MACROSTATE

↳ NUMBER OF MICROSTATES WHICH CORRESPOND TO IT.

↳ tells you everything there is to know about a system

↳ eg calc at what temp water freezes

→ $\underbrace{\text{patterns of ice formation}}_{\text{usually impossible}}$

For system we are talking about

$\Omega(N_\uparrow)$ = # of microstates of N spin system where number of up spin states is $N_\uparrow$

or $S_T = 2N_\uparrow - N$

Combinatorics, start with $N=2$

$$2^N = 4 \quad \text{MICROSTATES} \quad \left\{ \begin{array}{ll} \uparrow\uparrow & N_\uparrow = 2, \ S_T = 2 \\ \downarrow\uparrow & \\ \uparrow\downarrow & \Big\} \ N_\uparrow = 1, \ S_T = 1 \\ \downarrow\downarrow & N_\uparrow = 0 \quad S_T = 0 \end{array} \right\} \ 3 \ \text{MACROSTATE}$$

$$\Omega(N_\uparrow = 2) = \Omega(N_\uparrow = 0) = 1$$

$$\Omega(N_\uparrow = 1) = 2$$

$$(\uparrow + \downarrow)^N \quad \text{FOR } N=2 = (\uparrow + \downarrow)(\downarrow + \uparrow)$$

$$= \underbrace{\uparrow\uparrow}_{N_\uparrow = 2} + \underbrace{\uparrow\downarrow + \downarrow\uparrow}_{N_\uparrow = 1} + \underbrace{\downarrow\downarrow}_{N_\uparrow = 0}$$

Text book SHOWS

$$\Omega(N, n) = \frac{N!}{n!\,(N-n)!} = \binom{N}{n}$$

number of ways of choosing n things out of N

This immediately gives

$$\Omega(N_\uparrow) = \frac{N!}{(N-N_\uparrow)!\,N_\uparrow!}$$

$$= \frac{N!}{N_\downarrow!\,N_\uparrow!}$$

OUR treatment $(\uparrow + \downarrow)^N$ is a realization of

$$(a+b)^N = \sum_{k=0}^{N} \binom{N}{N-k} a^{N-k} b^k$$

BINOMIAL $a \to$ prob of $N-k$
$b \to$ prob of k

Allows us to write

$$(\uparrow + \downarrow)^N = \underbrace{\sum_{k=0} \binom{N}{N-k}}_{\substack{\text{MICROSTATES} \\ \text{WITH } N-k \text{ up}}} \underbrace{\uparrow \cdots \uparrow}_{\substack{N-k \\ \text{UP}}} \underbrace{\downarrow \downarrow \downarrow}_{\substack{k \\ \text{DOWN}}}$$

$$= \frac{N!}{(N-k)!\,k!}$$

TEXT BOOK ON COMBINATORICS

3 COINS

PENNY	NICKLE	DIME	
H	H	H	← MICROSTATE
H	H	T	TOTAL = 8
H	T	H	
T	H	H	
H	T	T	
T	T	H	
T	H	T	
T	T	T	

\# HEADS = MACROSTATE

MICROSTATE $\xrightarrow{\text{KNOW}}$ MACROSTATE

MACRO STATE $\not\longrightarrow$ MICROSTATE

\# MICROSTATES, SAME MACRO = MULTIPLICITY

3H → 1

2H → 3

1H → 3

PROB of ANY MACROSTATE $= \dfrac{\Omega(n)}{\Omega(N)}$

IF HAVE 100 COINS 2^{100} μ STATES

MACROSTATES 101

$$\left.\begin{array}{c} 0H \\ 1H \\ 2H \\ \vdots \\ 100H \end{array}\right\} 101$$

MULTIPLICITIES OF MACROSTATES

$0H \rightarrow T \cdots T \quad \Omega(0) = 1$

$$1H \rightarrow \left.\begin{array}{l} H\,T \cdots T \\ T\,H \cdots T \\ T\,T\,H \cdots T \end{array}\right\} \Omega(1) = 100$$

$\Omega(1)$ is number of ways of choosing to turn over one COIN

$\Omega(2)$ is number of ways of ~~turning~~ turning over 2 COINS

$\Omega(2)$ number of ways of turning over 2

100 CHOICES for FIRST

→ For each of these there are 99 choices for the second.

→ BUT in any pair could choose either order.

H T ---- T T ---- T

↓

H T ---- H T ---- T

this is the same as

T T ---- H T ---- T

↓

H T ---- H T ---- T

so $\Omega(2) = \dfrac{100 \cdot 99}{2}$

$\Omega(3) = \dfrac{100 \cdot 99 \cdot 98}{3 \cdot 2}$

$$\Omega(n) = \frac{\overset{\downarrow \text{numerator}}{100 \cdot 99 \cdot 98 \cdots (100-n+1)}}{\underset{\uparrow \text{denominator}}{n \cdots 2 \cdot 1}}$$

denom $= n!$

num $= \dfrac{100!}{(100-n)!}$

$$\Omega(n) = \frac{100!}{n!\,(100-n)!} = \binom{100}{n}$$

$$\Omega(N, n) = \frac{N!}{n!\,(N-n)!} = \binom{N}{n}$$